

Model Question – 10

Subject : Mathematics XII (Mat. 402/008)

Time : 3 hrs

F.M. 75

Attempt all the questions:

Group "A"

Rewrite the correct option in your answer sheet:

11X1=11

- How many numbers between 300 and 3000 can be formed with the digits 0, 1, 2, 3, 4 and 5 without repetitions?
a. 180 b. 195 c. 175 d. 200
- The value of k so that the equation $2x^2 - (5 + k)x + 8 = 0$ has roots numerically equal but opposite in sign is
a. k = -5 b. k = 5 c. k = 3 d. k = 8
- $\cos\left(\tan^{-1}\frac{3}{4}\right) =$
a. $\frac{4}{5}$ b. $\frac{3}{5}$ c. $\frac{5}{3}$ d. $\frac{5}{4}$
- Angle between the diagonals of a cube is
a. $\cos^{-1}\left(\frac{1}{3}\right)$ b. $\cos^{-1}\left(\frac{1}{2}\right)$
c. $\cos^{-1}\left(\pm\frac{1}{3}\right)$ d. $\cos^{-1}\left(\pm\frac{1}{2}\right)$
- The value of $|\vec{j} \times \vec{k} + \vec{j}(\vec{k} \times \vec{i}) + \vec{k}(\vec{j} \times \vec{i})|$ is
a. 0 b. 1 c. 2 d. 3
- The differential co-efficient of x^x is
a. $x^x \log x$ b. $x^x \left(\log x + \frac{1}{x}\right)$ c. $x^x (\log x + 1)$ d. $x^x - 1$
- The value of $\int \frac{dx}{1 - 2\cos x}$ is
a. $\frac{1}{\sqrt{3}} \log \left(\frac{\sqrt{3} \tan \frac{x}{2} - 1}{\sqrt{3} \tan \frac{x}{2} + 1} \right)$ b. $\frac{1}{\sqrt{3}} \log \left(\frac{1 + \sqrt{3} \tan \frac{x}{2}}{\sqrt{3} \tan \frac{x}{2} - 1} \right)$
c. $\sqrt{3} \log \left(\frac{\sqrt{3} \tan \frac{x}{2} - 1}{\sqrt{3} \tan \frac{x}{2} + 1} \right)$ d. $\sqrt{3} \log \left(\frac{1 + \sqrt{3} \tan \frac{x}{2}}{\sqrt{3} \tan \frac{x}{2} - 1} \right)$
- Intercepts made on the co-ordinate axes by the plane $2x - y + 2z = 4$ are
a. 2, 2, -4 b. 2, 2, 4 c. 2, -4, 2 d. 2, 2, 2

- 9) The product of two regression co-efficient is
 a. 1 b. -1 c. ≥ 1 d. ≤ 1
- 10) The system of equations:
 $x_1 + x_2 + x_3 = -3$; $3x_1 + x_2 + 2x_3 = -2$; $2x_1 + 4x_2 + 7x_3 = 7$
 a. consistent b. Inconsistent
 c. consistent and finite solutions d. consistent and infinite many solutions
- 11) If a body of mass 0.5 kg and initially at rest, is subjected to a force of 2N for 1 sec then the velocity acquired during the second is
 a. 4m/s b. 5 m/s c. 3 m/s d. 40 m/s

Group "B"

Short answer questions:

8X5=40

- 12) Show that: $\sum_{n=1}^{\infty} \frac{n^2}{(n+1)!} = e - 1$
- 13) Using De Moivre's theorem find the fourth root of $-\frac{1}{2} + \frac{\sqrt{3}}{2}i$
- 14) Find the direction cosines of the two lines which satisfy the relations
 $2l + 2m - n = 0$ and $lm + mn + nl = 0$.
- 15) a) Find the correlation co-efficient between the two variables:

No. of pair of observations
 Standard deviations

X	Y
10	10
2.05	2.41
42	58

Sum of the squares of deviations from their respective means

Sum of the products of deviations of X and Y from their respective means = 36

- b) A committee of 5 is to be formed out of 8 men and 6 women. Find the probability that in the committee there will be 3 men and 2 women.
- 16) Find from first principle, the derivative of $\sin(\log x)$.
- 17) Evaluate: $\int \frac{dx}{2\sin x + 3\cos x}$
- 18) Solve the following LP problem, using simplex method
 $2x + 3y \leq 24$
 $x + 2y \leq 4$
 $x, y \geq 0$

- 19) A stone of mass 1 kg falls from the top of a vertical cliff. After (i) falling for 3 seconds (ii) descending 800 cm, it reaches the foot of the cliff and penetrates 25 cms into the sand. Find the resistance offered by the sand. ($g = 9.8 \text{ m/s}^2$)

Group "C"

Long answer questions:

3×8=24

20)

- a) A committee is to be chosen from 12 men and 8 women and is to consist of 3 men and 2 women. How many such committee can be formed? 2
- b) Let $G = \{0, 1, 2\}$. Form a composition table for G under addition modulus 3. Find the inverse element of 2. 2
- c) If the quadratic equations $x^2 + qx + pr = 0$ and $x^2 + rx + pq = 0$ have one root common prove that: $p + q + r = 0$ 2
- d) Using the principle of mathematical induction, show that:

$$1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

21)

- a) Verify Lagrange's mean value theorem for the functions: $f(x) = x^3 + x^2 - 6x$ in $[-1, 4]$.
- b) Solve the differential equations: $(1 + x^2) \frac{dy}{dx} + y = e^{\tan^{-1} x}$

22)

- a) Find the angle between the two lines whose direction cosines are given by $4l + 3m - 2n = 0$ and $lm - mn + nl = 0$
- b) Find the equation of the plane through the intersection of the planes $x + y + z = 6$ and $2x + 3y + 4z + 5 = 0$ and perpendicular to the plane $4x + 5y - 3z = 8$.

Answers:

Group A

1) a	2) a	3) a	4) a
5) b	6) c	7) a	8) c
9) d	10) c	11) a	

Group B

13) $-1, \pm \left(\frac{\sqrt{3}}{2} + \frac{1}{2}i \right), \pm \left(\frac{\sqrt{3}}{2} - \frac{1}{2}i \right)$
14) $l = \frac{1}{3}, m = -\frac{2}{3}, n = -\frac{2}{3}$
15) (a) 0.73 (b) $\frac{60}{143}$
16) $\frac{1}{x} \cos(\log x)$
17) $\frac{1}{x} \log \frac{\sqrt{13} + 2 - \tan \frac{x}{2}}{\sqrt{13} - 2 + \tan \frac{x}{2}} + C$
18) Max.P = 30x + 20y, when x = 11, y = 2
19) 33 kg wt.

Group C

20) (a) 6160 (b) 1
21) (b) $y = \frac{1}{2} e^{\tan^{-1} s} + c e^{-\tan^{-1} s}$
22) a) $\cos^{-1} \frac{10}{39}$ b) $x + 7y + 13z + 96 = 0$