Model Quesiton – 4

Subject : Mathematics XII (Mat. 402/008)

Time : 3 hrs   **F.M. 75**

Attempt all the questions:

Group "A"

Rewrite the correct option in your answer sheet: **11X1=11**

1) In how many ways can the letters of the word ELEMENT be arranged?

   a. 640   b. 740   c. 840   d. 1040

2) The Euler form of the complex number $\sqrt{3} + i$ is

   a. $5e^{\frac{\pi}{6}i}$ b. $4e^{\frac{\pi}{6}i}$ c. $4e^{\frac{\pi}{3}i}$ d. $2e^{\frac{\pi}{6}i}$

3) What is the value of $\sin\left(\cos^{-1}\frac{3}{5}\right)$?

   a. $\frac{4}{5}$ b. $\frac{3}{5}$ c. $\frac{2}{5}$ d. $\frac{1}{5}$

4) The length of the latus rectum of the ellipse $\frac{x^2}{16} + \frac{y^2}{4} = 1$ is

   a. 16   b. 12   c. 4   d. 2

5) If $\vec{a} \cdot \vec{b} = 48,$ are $|\vec{a}| = 15$ and $|\vec{b}| = 4,$ then the value of $|\vec{a} \times \vec{b}|$ is

   a. 36   b. 32   c. 28   d. 24

6) If a line makes angles 60^0 and 45^0 with the positive x-axis and z – axis is respectively then the acute angle made by the line with positive y – axis is

   a. $\frac{\pi}{6}$ b. $\frac{\pi}{4}$ c. $\frac{\pi}{3}$ d. $\frac{\pi}{8}$

7) The mean and the variance of the binomial distribution are 6 and 4 respectilvey. What is the value of n?

   a. 12   b. 18   c.24   d. 30

8) The derivative of $\log \sin h \frac{x}{a}$ is

   a. $\cot h\frac{x}{a}$ b. $\frac{1}{a}\cot h\frac{x}{a}$ c. $\frac{1}{a}\tan h\frac{x}{a}$ d. $\tan h\frac{x}{a}$

9) The general solution of the differential equation $\dfrac{dy}{dx} = \dfrac{x}{y}$ is

    a. $x^2 + y^2 = c$ b. $x^2 - y^2 = c$ c. $x^2 + y^2 = 0$ d. $x^2 - y^2 = 0$

10) The puill of the earth on a body is 49N. if the acceleration due to gravity is 9.8 m/s2, then the mass of the body is

    a. 5 kg   b. 4 kg   c. 3 kg   d. 2 kg

OR

The demand and supply equations for a product are

    $P_d = 240 - 3Q^2$

and $P_s = 160 + 2Q^2$ respectively.

What is the market equilibrium price?

    a. 4   b. 100   c. 192   d.92

11) For what value of k does the equation $(3k+1)x^2 + 2(k+1)x + k = 0$ have reciprocal roots?

    a. $\dfrac{1}{2}$ b. $\dfrac{3}{2}$ c. $-\dfrac{3}{2}$ d. $-\dfrac{1}{2}$

Group "B"

Short answer questions: **8X5=40**

12) In how many ways can the letters of the word LOGIC be arranged so that no two vowels are together?

13) Find the sum of the first n terms of the series 1 + 3 + 3 6 + 10 + …………

14) Solve:

    a) $\tan^{-1}x - \cot^{-1}x = 0$ 2

    b) $2\cos^2 x + \sin x \cos x - \sin^2 x = 0$ 3

15) The regression coefficients of x on y and y on x are 0.84 and 0.32 respectively. If the arithmetic means of x and y series are 42 and 26 respectively, find two equations of lines of regression. Also find the value of y when x = 20   4+1=5

16) Find, from definition, the differential coefficient of $\tan\sqrt{x}$.

17) Evaluate: $\displaystyle\int \dfrac{dx}{3\sin x + 4\cos x}$

18) Solve the following system of equations using Gaussian elimination method:

    x+3y − z = -2

    3x + 2y − z = 3

    -6x − 4y − 2z = 18

19) If R be the horizontal range of a projectile and h its greatest height, prove that its initial velocity is $\sqrt{2g\left(h + \dfrac{R^2}{16h}\right)}$

OR

The inter-industry transaction table presented below was formed for an economy of two industries P and Q for a certain year

Producers	Users		Final	Total
	P	Q		
P	250	160	90	500
Q	200	120	80	400

Find the total output to be produced by P and Q when final demands are 152 of P and 114 untis of Q.

Group "C"

Long answer questions: 8×3=24

20)

    a) Apply De Moivres theorem to compute $(-1 + i)^{14}$.   4

    b) If the roots of the equation $lx^2 + nx + n = 0$ be in the ratio p:q, prove that:

$$\sqrt{\frac{p}{q}} + \sqrt{\frac{q}{p}} + \sqrt{\frac{n}{l}} = 0 \qquad 4$$

21)

    a) Show that $9x^2 + 4y^2 - 18x - 16y - 11 = 0$ represents the equation of an ellipse.

    b) Find the coordinate of the vertices and foci of $9x^2 + 4y^2 - 18x - 16y - 11 = 0$.

    c) Find the equation of the plane passing through the points (2, 2, 1) and   (9, 3, 6) and normal to the plane $2x + 6y + 6z = 9$ 1+3+4=8

22)

  a) Use vector method to prove that $\sin(A + B) = \sin A \cos B + \cos A \sin B$.

  b) Show that the set of all positive rational numbers form an abelian group under the composition defined by $a0b = \dfrac{ab}{2}$ 4+4=8

Answers:

Group A

1) (c)	2) (d)	3) (a)	4) (d)
5) (a)	6) (d)	7) (b)	8) (b)
9) (b)	10) (a) OR (a)	11) (d)	

Group B

12) 72		
13) $\dfrac{n(n+1)(n+2)}{6}$		
14)		

(a) $\dfrac{\pi}{4}$ (b) $n\pi + (-1)^n \dfrac{\pi}{6}$

15) y = 0.32x + 12
 	
     x = 0.84 y + 20.16
 	
     18.96

16) $\dfrac{\sec^2 \sqrt{x}}{2\sqrt{x}}$
17) $\dfrac{1}{5}\log\tan\left(\dfrac{x}{2} + \dfrac{1}{2}\tan^{-1}\dfrac{4}{3}\right) + C$
18) (1, -3, -6)
19) OR 800 units and 620 units

Group C

20) (a) 128 i

21)
 	
  (b) vertices = (1, 5) and (1, -1)
 	
       Foci = $\left(1, 2 \pm \sqrt{5}\right)$
 	
 (c) 3x + 4y − 5z = 9